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Active Control of a Plate Subjected to Parametric and External Force using Negative Velocity Feedback

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Abstract

The dynamical behavior of a Plate subjected to parametric and external excitations forces is investigated. The method of multiple time scale perturbation technique is applied to solve the non-linear differential equations describing the system up to the second order approximation. Resonance cases at this approximation are obtained and studied numerically to determine the worst resonance case. The effects of different parameters are studied. Stability of the steady state solution for the selected resonance case and frequency response equation are studied via Matlab 8.0 and Maple 16.

Keywords: Vibration Control, Resonance Cases, Multiple time Scale, Stability.

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التحكم النشط في صفيحة معرضة لقوى بارامترية وخارجية باستخدام التغذية الراجعة السلبية للسرعة

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الملخص:

تم دراسة السلوك الديناميكي لصفيحة معرضة لقوى بارامترية وخارجية. تم استخدام طريقة الأزمنة المضطربة المتعددة لحل المعادلات التفاضلية غير الخطية التي تصف النظام حتى التقريب من الدرجة الثانية. تم الحصول على حالات الرنين عند هذا التقريب ودراستها عدديا لتحديد أسوأ حالة رنين. تم دراسة تأثير البارامترات المختلفة. تم دراسة الحالة المستقرة لحالة الرنين المختارة ومعادلة الاستجابة الترددية باستخدام الماتلاب 8.0 والمابل 16.

الكلمات الدالة: التحكم في الاهتزاز، حالات الرنين، الأزمنة المضطربة المتعددة، الاستقرار.

1. Introduction

Chaos is one of the most exciting topics in the field of physical sciences. Researchers have determined the behavior of vibrations and control of both vibrations and chaos for various vibrating systems. Many ideas and approaches for controlling chaos have been proposed in the past twenty years [1-5].

Oueni et al. [6] studied a non-linear active vibration absorber coupled with the plant through user-defined cubic nonlinearities. Tondl et al. [7] studied a dynamic absorber, which can move in the transverse or longitudinal direction, which attached to an excited simple pendulum. Nayfeh et al. [8] discussed dynamics of machining using quadratic and cubic stiffness of machine tools, which accounts for the regenerative effects.

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Zhang [9] analyzed the global bifurcations and chaotic dynamics of a parametrically excited, simply supported rectangular thin plate. The method of multiple scales is used to obtain the averaged equations in the presence of 1:1 internal resonance and primary parametric resonance. Zhang et al. [10] investigated the local and global bifurcations of a parametrically and externally excited simply supported rectangular thin plate subjected to transversal and inplane excitation simultaneously.

Belhaq et al. [11] investigated the control of chaos of one-degree-of-freedom system with both quadratic and cubic nonlinearities subjected to combined parametric and external excitations. Glabisz [12] studied the stability of one-degree-of-freedom system under velocity and acceleration dependent non-conservative forces. Eissa and Amer [13] controlled the vibration of a second order system simulating the first mode of a cantilever beam subjected to primary and sub-harmonic resonance using cubic velocity feedback. El-Bassiouny [14] made an investigation on the control of the vibration of the crankshaft in internal combustion engines subjected to both external and parametric excitations via an elastomeric absorber having both quadratic and cubic stiffness nonlinearities.

Gao and Chen [15] studied active vibration control for a Bilinear system with nonlinear velocity time-delayed feedback. Yingli et al. [16] studied dynamic effects of delayed feedback control on nonlinear vibration floating raft systems. Amer et al. [17] studied vibration control of three degree of freedom parametrically excited cantilever beam. Samira et al [18] studied stability and control of non-linear dynamical system subjected to multi external force with

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velocity feedback. Samira et al [19] studied amplitude reduction of parametric resonance by velocity feedback control.

The objective of this work is to study dynamical behavior of a plate subject to parametric and external excitation forces under state feedback active control. The method of multiple scale perturbation technique is applied to obtain the solution up to the second order approximation.

2. Mathematical Analysis:

The equation of the dynamical behavior of a plate subject to parametric and external excitation forces is given by:

$$\ddot{x} + \varepsilon\mu_1\dot{x} + \omega_1^2 x + \varepsilon\beta_1\dot{x}\dot{y} + \varepsilon\beta_2\dot{y}^2 + \varepsilon\gamma_1 x\dot{y}^2 + 2\varepsilon\gamma_3\dot{x}y\dot{y} + \varepsilon\gamma_4 y\dot{y}^2 = \varepsilon f_1 x\cos\Omega t + R_1 \quad (1)$$

$$\ddot{y} + \varepsilon\mu_2\dot{y} + \omega_2^2 y + \varepsilon\beta_3\dot{x}^2 + \varepsilon\gamma_2\dot{x}^2 y = \varepsilon f_2\cos\Omega t + R_2 \quad (2)$$

where $R_1 = -\varepsilon G_1\dot{x}^3$, $R_2 = -\varepsilon G_2\dot{y}^3$, and x, y are the vibration amplitudes of the composite laminated rectangular thin plate for the first-order and the second-order modes, respectively, μ_1 and μ_2 the modal damping coefficients, ω_1 and ω_2 the linear natural frequencies of the thin Plate, and Ω the excitation frequencies. f_1 and f_2 the excitation forces, β_i, γ_j, $(i = 1,2,3,\ \ j = 1,2,3,4)$ are non-linear coefficients, and G_1, G_2 are gain coefficients. We seek a second order uniform expansion for the solutions of equation (1) and (2) in the form:

$$x(t,\varepsilon) = x_0(T_0,T_1) + \varepsilon x_1(T_0,T_1) + \varepsilon^2 x_2(T_0,T_1) + O(\varepsilon^3) \quad (3)$$

$$y(t,\varepsilon) = y_0(T_0,T_1) + \varepsilon y_1(T_0,T_1) + \varepsilon^2 y_2(T_0,T_1) + O(\varepsilon^3) \quad (4)$$

where $T_n = \varepsilon^n t \quad ,(n = 0,1,2)$

and the time derivatives became

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \ldots\ , \qquad \frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 D_0^2 + \ldots,$$

(5)

where and ε is small a perturbation parameter and $0 < \varepsilon << 1$, T_0 is the fast time scale, T_1 is the slow time scales. Substituting equations (3), (4) and (5) into equations (1) and (2) and equating the

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coefficients of same power of ε in both sides, we obtain the following set of ordinary differential equations:

Order ε^0:

$$\left(D_0^2+\omega_1^2\right)x_0=0$$

(6)

$$\left(D_0^2+\omega_2^2\right)y_0=0$$

(7)

Order ε^1:

$$\left(D_0^2+\omega_1^2\right)x_1=-2D_0D_1x_0-\mu_1D_0x_0-\beta_1D_0x_0D_0y_0-\beta_2(D_0y_0)^2-\gamma_1x_0(D_0y_0)^2$$
$$-2\gamma_3y_0D_0x_0D_0y_0-\gamma_4y_0(D_0y_0)^2+f_1x_0\cos\Omega T_0-G_1\left(D_0x_0\right)^3$$

(8)

$$\left(D_0^2+\omega_2^2\right)y_1=-2D_0D_1y_0-\mu_2D_0y_0-\beta_3(D_0x_0)^2-\gamma_2y_0(D_0x_0)^2+f_2\cos\Omega T_0$$
$$-G_2\left(D_0y_0\right)^3$$

(9)

Order ε^2:

$$\left(D_0^2+\omega_1^2\right)x_2=-D_1^2x_0-2D_0D_2x_0-2D_0D_1x_1-\mu_1D_1x_0-\mu_1D_0x_1-\beta_1D_0x_0D_1y_1$$
$$-\beta_1D_0x_0D_0y_1-\beta_1D_1x_0D_0y_1-\beta_1D_0x_1D_0y_0-2\beta_2D_0y_0D_1y_0-2\beta_2D_0y_0D_0y_1$$
$$-2\gamma_1x_0D_0y_0D_1y_0-2\gamma_1x_0D_0y_0D_0y_1-\gamma_1x_1(D_0y_0)^2-2\gamma_3y_0D_0x_0D_1y_0$$
$$-2\gamma_3y_0D_0x_0D_0y_1-2\gamma_3y_0D_1x_0D_0y_0-2\gamma_3y_0D_0x_1D_0y_0-2\gamma_3y_1D_0x_0D_0y_0$$
$$-2\gamma_4y_0D_0y_0D_1y_0-2\gamma_4y_0D_0y_0D_0y_1-\gamma_4y_1(D_0y_0)^2+f_1x_0\cos\Omega T$$
$$-3G_1\left(D_0x_0\right)^2D_1x_0-3G_1\left(D_0x_0\right)^2D_0x_1$$

(10)

$$\left(D_0^2+\omega_2^2\right)y_2=-D_1^2y_0-2D_0D_2y_0-2D_0D_1y_1-\mu_2D_1y_0-\mu_2D_0y_1-2\beta_3D_0x_0D_1x_0$$
$$-2\beta_3D_0x_0D_0x_1-2\gamma_2y_0D_0x_0D_1x_0-2\gamma_2y_0D_0x_0D_0x_1-\gamma_2y_1\left(D_0x_0\right)^2$$
$$-3G_2\left(D_0y_0\right)^2D_1y_0-3G_2\left(D_0y_0\right)^2D_0y_1$$

(11)

The general solution of equations (6) and (7) is given by

$$x_0(T_0,T_1)=A_0(\mathrm{T}_1)\exp(i\,\omega_1T_0)+\bar{A}_0(\mathrm{T}_1)\exp(-i\,\omega_1T_0)$$

(12)

$$y_0(T_0,\mathrm{T}_1)=B_0(\mathrm{T}_1)\exp(i\,\omega_2T_0)+\bar{B}_0(\mathrm{T}_1)\exp(-i\,\omega_2T_0)$$

(13)

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where A_0, B_0 are unknown functions in T_1 at this level of approximation and can be determined by elimination the secular terms from the next order of perturbation. Substituting equations (12) and (13) into equations (8), (9) yields

$$\left(D_0^2+\omega_1^2\right)x_1=(-2i\,\omega_1 D_1 A_0-\mu_1 i\,\omega_1 A_0-2\gamma_1\omega_2^2 A_0\,B_0\bar{B}_0-3i\,\omega_1^3 A_0^2\,\bar{A}G_1)\exp(i\,\omega_1 T_0)$$
$$+\beta_2\omega_2^2\,B_0^2\exp(2i\,\omega_2 T_0)+\beta_1\omega_1\omega_2 A_0\,B_0\exp(i\,(\omega_1+\omega_2)T_0)-\beta_1\omega_1\omega_2 A_0\,\bar{B}_0$$
$$\cdot\exp(i\,(\omega_1-\omega_2)T_0)+(\gamma_1\omega_2^2 A_0\,B_0^2+2\gamma_3\omega_1\omega_2 A_0\,B_0^2)\exp(i\,(\omega_1+2\omega_2)T_0)$$
$$+(\gamma_1\omega_2^2 A_0\,\bar{B}_0^2-2\gamma_3\omega_1\omega_2 A_0\,\bar{B}_0^2)\exp(i\,(\omega_1-2\omega_2)T_0)+\gamma_4\omega_2^2\,B_0^3\exp(3i\,\omega_2 T_0)$$
$$+G_1 i\,\omega_1^3 A_0^3\exp(3i\,\omega_1 T_0)-\gamma_4\omega_2^2\,\bar{B}_0 B_0^2\exp(i\,\omega_2 T_0)+\frac{f_1}{2}A_0\exp(i\,(\omega_1+\Omega)T_0)$$
$$+\frac{f_1}{2}A_0\exp(i\,(\omega_1-\Omega)T_0)-\beta_2\omega_2^2\,B_0\bar{B}_0+cc$$
(14)

$$\left(D_0^2+\omega_2^2\right)y_1=(-2i\,\omega_2 D_1\,B_0-\mu_2 i\,\omega_2\,B_0-3i\,\omega_2^3 B_0^2\,\bar{B}_0 G_2-2\omega_1^2\gamma_2 A_0\bar{A}_0\,B_0)$$
$$\cdot\exp(i\,\omega_2 T_0)+\beta_3\omega_1^2 A_0^2\exp(2i\,\omega_1 T_0)+G_2 i\,\omega_2^3 B_0^3\exp(3i\,\omega_2 T_0)$$
$$+\gamma_2\omega_1^2 A_0^2 B_0\exp(i\,(2\omega_1+\omega_2)T_0)+\gamma_2\omega_1^2 A_0^2\,\bar{B}_0\exp(i\,(2\omega_1-\omega_2)T_0)$$
$$+\frac{f_2}{2}\exp(i\,\Omega T_0)-\beta_3\omega_1^2 A_0\bar{A}_0+cc \qquad (15)$$

The general solutions of equations (14) and (15) are:

$$x_1(T_0,T_1)=A_1(T_1)\exp(i\,\omega_1 T_0)+E_1\exp(2i\,\omega_2 T_0)+E_2\exp(i\,(\omega_1+\omega_2)T_0)$$
$$+E_3\exp(i\,(\omega_1-\omega_2)T_0)+E_4\exp(i\,(\omega_1+2\omega_2)T_0)+E_5\exp(i\,(\omega_1-2\omega_2)T_0)$$
$$+E_6\exp(3i\,\omega_2 T_0)+E_7\exp(3i\,\omega_1 T_0)+E_8\exp(i\,\omega_2 T_0)+E_9\exp(i\,(\omega_1+\Omega)T_0)$$
$$+E_{10}\exp(i\,(\omega_1-\Omega)T_0)+E_{11}+cc$$
(16)

$$y_1(T_0,T_1)=B_1(T_1)\exp(i\,\omega_2 T_0)+E_{12}\exp(2i\,\omega_1 T_0)+E_{13}\exp(3i\,\omega_2 T_0)$$
$$+E_{14}\exp(i\,(2\omega_1+\omega_2)T_0)+E_{15}\exp(i\,(2\omega_1-\omega_2)T_0)+E_{16}\exp(i\,\Omega T_0)$$
$$+E_{17}+cc$$
(17)

Substituting equations (12), (13), (16) and (17) into equations (10), (11) and solving the resulting equation we get:

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$$x_2(T_0,T_1)=A_2(T_1)\exp(i\,\omega_1T_0)+E_{18}\exp(i\,\omega_2T_0)+E_{19}\exp(i\,(\omega_1+\omega_2)T_0)$$
$$+E_{20}\exp(i\,(\omega_1-\omega_2)T_0)+E_{21}\exp(2i\,\omega_1T_0)+E_{22}\exp(2i\,\omega_2T_0)$$
$$+E_{23}\exp(3i\,\omega_1T_0)+E_{24}\exp(3i\,\omega_2T_0)+E_{25}\exp(i\,(\omega_1+2\omega_2)T_0)$$
$$+E_{26}\exp(i\,(\omega_1-2\omega_2)T_0)+E_{27}\exp(i\,(3\omega_1+\omega_2)T_0)+E_{28}$$
$$\cdot\exp(i\,(3\omega_1-\omega_2)T_0)+E_{29}\exp(i\,(\omega_1+3\omega_2)T_0)+E_{30}\exp(i\,(\omega_1-3\omega_2)T_0)$$

$$+E_{31}\exp(i\,(\omega_1+\Omega)T_0)+E_{32}\exp(i\,(\omega_1-\Omega)T_0)+E_{33}\exp(i\,(2\omega_1+\omega_2)T_0)$$
$$+E_{34}\exp(i\,(2\omega_1-\omega_2)T_0)+E_{35}\exp(i\,(2\omega_1+2\omega_2)T_0)+E_{36}\exp(i\,(2\omega_1-2\omega_2)T_0)$$
$$+E_{37}\exp(i\,(\omega_1+4\omega_2)T_0)+E_{38}\exp(i\,(\omega_1-4\omega_2)T_0)+E_{39}\exp(i\,(3\omega_1+2\omega_2)T_0)$$
$$+E_{40}\exp(i\,(3\omega_1-2\omega_2)T_0)+E_{41}\exp(4i\,\omega_2T_0)+E_{42}\exp(5i\,\omega_2T_0)$$
$$+E_{43}\exp(i\,(\omega_1+\omega_2+\Omega)T_0)+E_{44}\exp(i\,(\omega_1-\omega_2+\Omega)T_0)$$
$$+E_{45}\exp(i\,(\omega_1+\omega_2-\Omega)T_0)+E_{46}\exp(i\,(\omega_1-\omega_2-\Omega)T_0)+E_{47}$$
$$\cdot\exp(i\,(\omega_2+\Omega)T_0)+E_{48}\exp(i\,(\omega_2-\Omega)T_0)+E_{49}\exp(i\,(\omega_1+2\omega_2+\Omega)T_0)$$
$$+E_{50}\exp(i\,(\omega_1-2\omega_2+\Omega)T_0)+E_{51}\exp(i\,(\omega_1+2\Omega)T_0)+E_{52}$$
$$\cdot\exp(i\,(\omega_1-2\Omega)T_0)+E_{53}\exp(i\,(\omega_1+2\omega_2-\Omega)T_0)+E_{54}\exp(i\,(\omega_1-2\omega_2-\Omega)T_0)$$
$$+E_{55}\exp(i\,(2\omega_1+3\omega_2)T_0)+E_{56}\exp(i\,(2\omega_1-3\omega_2)T_0)+E_{57}\exp(i\,(2\omega_2+\Omega)T_0)$$
$$+E_{58}\exp(i\,(2\omega_2-\Omega)T_0)+E_{59}\exp(i\,\Omega T_0)+E_{60}\exp(i\,(3\omega_2+\Omega)T_0)+E_{61}$$
$$\cdot\exp(i\,(3\omega_2-\Omega)T_0)+E_{62}\exp(i\,(3\omega_1+\Omega)T_0)+E_{63}\exp(i\,(3\omega_1-\Omega)T_0)$$
$$+E_{64}\exp(5i\,\omega_1T_0)+E_{65}+cc \tag{18}$$
$$y_2(T_0,T_1)=B_2(T_1)\exp(i\,\omega_2T_0)+E_{66}\exp(2i\,\omega_1T_0)+E_{67}\exp(2i\,\omega_2T_0)$$

$$+E_{68}\exp(3i\,\omega_2T_0)+E_{69}\exp(i\,(2\omega_1+\omega_2)T_0)+E_{70}\exp(i\,(2\omega_1-\omega_2)T_0)$$

$$+E_{71}\exp(i\,\Omega T_0)+E_{72}\exp(i\,(\omega_1+2\omega_2)T_0)+E_{73}\exp(i\,(\omega_1-2\omega_2)T_0)$$
$$+E_{74}\exp(i\,(2\omega_1+2\omega_2)T_0)+E_{75}\exp(i\,(2\omega_1-2\omega_2)T_0)+E_{76}\exp(i\,(\omega_1+3\omega_2)T_0)$$
$$+E_{77}\exp(i\,(\omega_1-3\omega_2)T_0)+E_{78}\exp(4i\,\omega_1T_0)+E_{79}\exp(i\,(\omega_1+\omega_2)T_0)+E_{80}$$
$$\cdot\exp(i\,(\omega_1-\omega_2)T_0)+E_{81}\exp(i\,(2\omega_1+\Omega)T_0)+E_{82}\exp(i\,(2\omega_1-\Omega)T_0)$$

$$+E_{83}\exp(5i\,\omega_2T_0)+E_{84}\exp(i\,(\omega_1+4\omega_2)T_0)+E_{85}\exp(i\,(\omega_1-4\omega_2)T_0)+E_{86}$$
$$\cdot\exp(i\,(2\omega_1+3\omega_2)T_0)+E_{87}\exp(i\,(2\omega_1-3\omega_2)T_0)+E_{88}\exp(i\,(4\omega_1+\omega_2)T_0)$$
$$+E_{89}\exp(i\,(4\omega_1-\omega_2)T_0)+E_{90}\exp(i\,\omega_1T_0)+E_{91}\exp(i\,(2\omega_1+\omega_2+\Omega)T_0)$$

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$$+E_{92}\exp(i\,(2\omega_1-\omega_2+\Omega)T_0)+E_{93}\exp(i\,(2\omega_1+\omega_2-\Omega)T_0)$$

$$+E_{94}\exp(i\,(2\omega_1-\omega_2-\Omega)T_0)+E_{95}\exp(i\,(\omega_2+\Omega)T_0)+E_{96}\exp(i\,(\omega_2-\Omega)T_0)$$
$$+E_{97}\exp(i\,(2\omega_2+\Omega)T_0)+E_{98}\exp(i\,(2\omega_2-\Omega)T_0)+E_{99}+cc$$
(19)

where $E_n,(\mathrm{n}=1,\ldots,99)$ are complex functions in T_1 and cc denotes the complex conjugate terms. From the above derived solutions, the reported resonance cases are:

1) Primary resonance: $\Omega\cong\omega_1,\Omega\cong\omega_2$.

2) Sub-harmonic resonance: $\Omega\cong2\omega_1,\ \Omega\cong3\omega_1,\ \Omega\cong2\omega_2,\ \Omega\cong3\omega_2$.

3) Super-harmonic resonance: $\Omega\cong\frac{1}{2}\omega_1$.

4) Internal resonance:
$2\omega_1\cong3\omega_2,\ 3\omega_1\cong2\omega_2,\ \omega_1\cong s_1\omega_2,\ \omega_2\cong s_2\omega_1,\ s_1=1,2,3,4,$ and
$s_2=2,3,4.$

5) Combined resonance:
$$\omega_1\cong\frac{1}{2}(\Omega+\omega_2),\ \omega_1\cong\pm\frac{1}{2}(\Omega-\omega_2),\ \omega_1\cong(\Omega+2\omega_2),\ \omega_1\cong\pm(\Omega-2\omega_2),$$

$$\omega_1\cong(\Omega+\omega_2),\ \omega_1\cong\pm(\Omega-\omega_2).$$

6) Simultaneous resonance: any combination of above resonance cases is considered as simultaneous resonance.

3. Stability analysis:

From the numerical solution which obtained that the worst resonance case is the simultaneous resonance case $\Omega=2\omega_1,\ \omega_2=\omega_1$. We introduce the detuning parameters σ_1 and σ_2 according to
$$\Omega=2\omega_1+\varepsilon\sigma_1,\ \ \omega_2=\omega_1+\varepsilon\sigma_2.$$
(20)

Substituting equation (20) into equations (14) and (15) and eliminating the secular and small divisor terms from x_1 and y_1 we get the following:
$$2i\,\omega_1D_1A_0=-\mu_1 i\,\omega_1A_0-2\omega_2^2\gamma_1A_0B_0\bar{B}_0-3i\,\omega_1^3A_0^2\bar{A}_0G_1+\gamma_1\omega_2^2B_0^2\bar{A}_0\exp(2i\,\sigma_2T_1)$$
$$-\gamma_4\omega_2^2B_0^2\bar{B}_0\exp(i\,\sigma_2T_1)-2\omega_1\omega_2\gamma_3\bar{A}_0B_0^2\exp(2i\,\sigma_2T_1)+\frac{f_1}{2}\bar{A}_0\exp(i\,\sigma_1T_1)$$

(21)

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$$2i\,\omega_2 D_1 B_0 = -\mu_2 i\,\omega_2 B_0 - 2\gamma_2\omega_1^2 A_0\bar{A}_0 B_0 - 3i\,\omega_2^2 B_0^2\bar{B}_0 G_2 + \gamma_2\omega_1^2 A_0^2\bar{B}_0$$

$$\cdot\exp(-2i\,\sigma_2 T_1) \qquad (22)$$

We express the complex function A_0, B_0 in the polar form as

$$A_0(T_1) = \frac{1}{2}a(T_1)\exp(i\,\theta_1(T_1)), \qquad B_0(T_1) = \frac{1}{2}b(T_1)\exp(i\,\theta_2(T_1)) \qquad (23)$$

where a, b, θ_1 and θ_2 are real.

Substituting equation (23) into equations (21) and (22) and separating real and imaginary part yields:

$$a' = -\frac{1}{2}\mu_1 a - \frac{3}{8}\omega_1^2 a^3 G_1 + \frac{1}{8\omega_1}\omega_2^2\gamma_1 b^2 a\sin 2\varphi_2 - \frac{1}{4}\omega_2\gamma_3 a b^2\sin 2\varphi_2$$

$$-\frac{1}{8\omega_1}\omega_2^2\gamma_4 b^3\sin\varphi_2 + \frac{f_1}{4\omega_1}a\sin\varphi_1$$

(24)

$$a\theta_1' = \frac{1}{4\omega_1}\omega_2^2\gamma_1 b^2 a - \frac{1}{8\omega_1}\omega_2^2\gamma_1 b^2 a\cos 2\varphi_2 + \frac{1}{4}\omega_2\gamma_3 a b^2\cos 2\varphi_2$$

$$+\frac{1}{8\omega_1}\omega_2^2\gamma_4 b^3\cos\varphi_2 - \frac{f_1}{4\omega_1}a\cos\varphi_1$$

(25)

$$b' = -\frac{1}{2}\mu_2 b - \frac{3}{8}\omega_2^2 b^3 G_2 - \frac{1}{8\omega_2}\omega_1^2\gamma_2 a^2 b\sin 2\varphi_2$$

(26) $$b\theta_2' = \frac{1}{4\omega_2}\omega_1^2\gamma_2 a^2 b - \frac{1}{8\omega_2}\omega_1^2\gamma_2 a^2 b\cos 2\varphi_2$$

(27)

where $\varphi_1 = \sigma_1 T_1 - 2\theta_1,\ \varphi_2 = \theta_2 + \sigma_2 T_1 - \theta_1$.

For the steady state solution $a' = b' = 0$, $\varphi_m' = 0;\ m = 1, 2$. Then it follows from equations (24)-(27) that the steady state solutions are given by

$$0 = -\frac{1}{2}\mu_1 a - \frac{3}{8}\omega_1^2 a^3 G_1 + \left(\frac{1}{8\omega_1}\omega_2^2\gamma_1 b^2 a - \frac{1}{4}\omega_2\gamma_3 a b^2\right)\sin 2\varphi_2 - \frac{1}{8\omega_1}\omega_2^2\gamma_4 b^3\sin\varphi_2$$

$$+\frac{f_1}{4\omega_1}a\sin\varphi_1$$

(28)

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$$a\sigma_1 = \frac{1}{2\omega_1}\omega_2^2\gamma_1 b^2 a - \frac{1}{4\omega_1}\omega_2^2\gamma_1 b^2 a\cos 2\varphi_2 + \frac{1}{2}\omega_2\gamma_3 ab^2\cos 2\varphi_2$$

$$+\frac{1}{4\omega_1}\omega_2^2\gamma_4 b^3\cos\varphi_2 - \frac{f_1}{2\omega_1}a\cos\varphi_1$$

(29)

$$0 = -\frac{1}{2}\mu_2 b - \frac{3}{8}\omega_2^2 b^3 G_2 - \frac{1}{8\omega_2}\omega_1^2\gamma_2 a^2 b\sin 2\varphi_2$$

(30) $$b(\frac{1}{2}\sigma_1 - \sigma_2) = \frac{1}{4\omega_2}\omega_1^2\gamma_2 a^2 b - \frac{1}{8\omega_2}\omega_1^2\gamma_2 a^2 b\cos 2\varphi_2$$

(31)

From equations (28)-(31), we have the following cases:

Case 1: $a \neq 0$ and $b = 0$: in this case, the frequency response equation is given by:

$$(\frac{9}{64}\omega_1^4 G_1^2)a^6 + (\frac{3}{8}\mu_1\omega_1^2 G_1 - \frac{3}{16}f_1\omega_1 G_1\sin\varphi_1)a^4 + (\frac{1}{4}\mu_1^2 + \frac{f_1^2}{16\omega_1^2} - \frac{f_1}{4\omega_1}\mu_1\sin\varphi_1$$

$$+\frac{1}{4}\sigma_1^2 + \frac{f_1}{4\omega_1}\sigma_1\cos\varphi_1)a^2 = 0$$

(32)

Case 2: $a = 0$ and $b \neq 0$: in this case, the frequency response equation is given by:

$$(\frac{9}{64}\omega_2^4 G_2^2)b^6 + (\frac{3}{8}\mu_2\omega_2^2 G_2)b^4 + (\frac{1}{4}\mu_2^2 + (\frac{1}{2}\sigma_1 - \sigma_2)^2)b^2 = 0$$

(33)

Case 3: $a \neq 0$ and $b \neq 0$: in this case, the frequency response equation is given by the following equations:

$$(\frac{9}{64}\omega_1^4 G_1^2)a^6 + (\frac{3}{8}\mu_1\omega_1^2 G_1 - \frac{3}{16}f_1\omega_1 G_1\sin\varphi_1)a^4 + (\frac{3}{32}\omega_1\omega_2^2\gamma_4 G_1 b^3\sin\varphi_2)a^3$$

$$+(\frac{1}{4}\mu_1^2 + \frac{3}{64\omega_1^2}\omega_2^4\gamma_1^2 b^4 + \frac{f_1^2}{16\omega_1^2} - \frac{1}{16}\omega_2^2\gamma_3^2 b^4 + \frac{1}{16\omega_1}\omega_2^3\gamma_1\gamma_3 b^4 - \frac{f_1}{4\omega_1}\mu_1\sin\varphi_1$$

$$+\frac{1}{4}\sigma_1^2 - \frac{1}{4\omega_1}\omega_2^2\gamma_1\sigma_1 b^2 - \frac{f_1}{8\omega_1^2}\omega_2^2\gamma_1 b^2\cos\varphi_1 + \frac{f_1}{4\omega_1}\sigma_1\cos\varphi_1)a^2 + (\frac{1}{8\omega_1}\omega_2^2\gamma_4 b^3\mu_1$$

$$\cdot\sin\varphi_2 - \frac{f_1}{16\omega_1^2}\omega_2^2\gamma_4 b^3\sin\varphi_1\sin\varphi_2 + \frac{1}{16\omega_1^2}\omega_2^4\gamma_1\gamma_4 b^5\cos\varphi_2 - \frac{1}{8\omega_1}\omega_2^2\sigma_1\gamma_4 b^3\cos\varphi_2$$

$$-\frac{f_1}{16\omega_1^2}\omega_2^2\gamma_4 b^3\cos\varphi_1\cos\varphi_2)a + \frac{1}{64\omega_1^2}\omega_2^4\gamma_4^2 b^6 = 0$$

(34)

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and

$$(\frac{9}{64}\omega_2^4 G_2^2)b^6 + (\frac{3}{8}\mu_2\omega_2^2 G_2)b^4 + (\frac{1}{4}\mu_2^2 + \frac{3}{64\omega_2^2}\omega_1^4\gamma_2^2 a^4 + (\frac{1}{2}\sigma_1 - \sigma_2)^2 - \frac{1}{2\omega_2}\omega_1^2\gamma_2 a^2(\frac{1}{2}\sigma_1 - \sigma_2))b^2 = 0 \qquad (35)$$

3.1 Linear solution

Now to the stability of the linear solution of the obtained fixed let us consider A_0 and B_0 in the forms

$$A_0(T_1) = \frac{1}{2}(p_1 - iq_1)\exp(i\delta_1 T_1) \text{ and } B_0(T_1) = \frac{1}{2}(p_2 - iq_2)\exp(i\delta_2 T_1) \qquad (36)$$

where p_1, p_2, q_1 and q_2 are real values and considering $\delta_1 = \frac{1}{2}\sigma_1, \delta_2 = \sigma_2$.

Substituting equation (36) into the linear parts of equations (21), (22) and separating real and imaginary parts, the following system of equations are obtained:

Case 1: for the solution ($a \neq 0$ and $b = 0$), we get

$$p_1' + \frac{1}{2}\mu_1 p_1 + (\frac{1}{2}\sigma_1 - \frac{f_1}{4\omega_1})q_1 = 0 \qquad (37)$$

$$q_1' + (-\frac{1}{2}\sigma_1 - \frac{f_1}{4\omega_1})p_1 + \frac{1}{2}\mu_1 q_1 = 0 \qquad (38)$$

The stability of the linear solution is obtained from the zero-characteristic equation

$$\begin{vmatrix} -(\lambda + \frac{1}{2}\mu_1) & (-\frac{1}{2}\sigma_1 + \frac{f_1}{4\omega_1}) \\ (\frac{1}{2}\sigma_1 + \frac{f_1}{4\omega_1}) & -(\lambda + \frac{1}{2}\mu_1) \end{vmatrix} = 0 \qquad (39)$$

where $\lambda_{1,2} = -\frac{1}{2}\mu_1 \pm \frac{1}{4\omega_2}\sqrt{f_1^{\,2} - 4\omega_2^2\sigma_1^2}$

since μ_1 is positive then the solutions are stable.

Case 2: for the solution ($a = 0$ and $b \neq 0$), we get

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$$p_2' + \frac{1}{2}\mu_2 p_2 + \sigma_2 q_2 = 0$$

(40)

$$q_2' - \sigma_2 p_2 + \frac{1}{2}\mu_2 q_2 = 0$$

(41)

The stability of the linear solution is obtained from the zero-characteristic equation

$$\begin{vmatrix} -(\lambda + \frac{1}{2}\mu_2) & -\sigma_2 \\ \sigma_2 & -(\lambda + \frac{1}{2}\mu_2) \end{vmatrix} = 0$$

(42)

where $\lambda_{1,2} = -\frac{1}{2}\mu_2 \pm i\,\sigma_2$

since μ_1 is positive then the solutions are stable.

Case 3: for the solution ($a \neq 0$ and $b \neq 0$)we get

$$p_1' + \frac{1}{2}\mu_1 p_1 + (\frac{1}{2}\sigma_1 - \frac{f_1}{4\omega_1})q_1 = 0$$

(43)

$$q_1' + (-\frac{1}{2}\sigma_1 - \frac{f_1}{4\omega_1})p_1 + \frac{1}{2}\mu_1 q_1 = 0$$

(44)

$$p_2' + \frac{1}{2}\mu_2 p_2 + \sigma_2 q_2 = 0$$

(45) $q_2' - \sigma_2 p_2 + \frac{1}{2}\mu_2 q_2 = 0$

(46)

The stability of the linear solution in this case is obtained from the zero-characteristic equation

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$$\begin{vmatrix} -(\lambda+\frac{1}{2}\mu_1) & (-\frac{1}{2}\sigma_1+\frac{f_1}{4\omega_1}) & 0 & 0 \\ (\frac{1}{2}\sigma_1+\frac{f_1}{4\omega_1}) & -(\lambda+\frac{1}{2}\mu_1) & 0 & 0 \\ 0 & 0 & -(\lambda+\frac{1}{2}\mu_2) & -\sigma_2 \\ 0 & 0 & \sigma_2 & -(\lambda+\frac{1}{2}\mu_2) \end{vmatrix}=0$$

(47)

after extract we obtain that

$$\lambda^4+r_1\lambda^3+r_2\lambda^2+r_3\lambda+r_4=0,$$

(48)

where $r_1=\mu_1+\mu_2$, $r_2=\sigma_2^2+\frac{1}{4}\sigma_1^2-\frac{f_1^{\,2}}{16\omega_1^2}+\frac{1}{4}\mu_1^2+\mu_1\mu_2+\frac{1}{4}\mu_2^2$,

$$r_3=\frac{1}{4}\mu_1\mu_2(\mu_1+\mu_2)+\frac{1}{4}\sigma_1^2\mu_2+\mu_1\sigma_2^2-\frac{f_1^{\,2}}{16\omega_1^2}\mu_2,$$

$$r_4=(\sigma_2^2+\frac{1}{4}\mu_2^2)(\frac{1}{4}\mu_1^2+\frac{1}{4}\sigma_1^2-\frac{f_1^{\,2}}{16\omega_1^2})$$

According to the Routh-Huriwitz criterion, the above linear solution is stable if the following are satisfied:

$$r_1>0,\ r_1r_2-r_3>0,\ r_3(r_1r_2-r_3)-r_1^{\,2}r_4>0,\ r_4>0.$$

3.2 Non-linear solution

To determine the stability of the fixed points, one lets

$a=a_{10}+a_{11},\ b=b_{10}+b_{11}$ and $\varphi_m=\varphi_{m0}+\varphi_{m1},(m=1,2),$

(49)

where a_{10}, b_{10} and φ_{m0} are the solutions of equations (28-31) and $a_{11}, b_{11}, \varphi_{m1}$ are perturbations which are assumed to be small compared to a_{10}, b_{10} and φ_{m0}. Substituting equation (49) into equations (24-27), using equations (28-31) and keeping only the linear terms in $a_{11}, b_{11}, \varphi_{m1}$ we obtain:

$$a_{11}'=-(\frac{1}{2}\mu_1+\frac{9}{8}\omega_1^3\mathrm{G}_1\mathrm{a}_{10}^2+\frac{\gamma_3}{4}\omega_2\,\mathrm{b}_{10}^2\sin 2\varphi_{20}-\frac{f_1}{4\omega_1}\sin\varphi_{10})a_{11}-(\frac{2}{4}\omega_2\gamma_3 a_{10}\,\mathrm{b}_{10}\sin 2\varphi_{20}$$

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$$-\frac{2}{8\omega_1}\omega_2^2\gamma_1 b_{10}\sin 2\varphi_{20}+\frac{3}{8\omega_1}\omega_2^2\gamma_4 b_{10}^2\sin\varphi_{20})b_{11}+(\frac{2}{8\omega_1}\omega_2^2\gamma_1 b_{10}^2\cos 2\varphi_{20}$$

$$-\frac{2}{4}\omega_2\gamma_3 a_{10} b_{10}^2\cos 2\varphi_{20}-\frac{1}{8\omega_1}\omega_2^2\gamma_4 b_{10}^3\cos\varphi_{20})\varphi_{21}+(\frac{f_1}{4\omega_1}a_{10}\cos\varphi_{10})\varphi_{11} \tag{50}$$

$$\varphi_{11}^{'}=(\frac{\sigma_1}{a_{10}}-\frac{\gamma_1}{2\omega_1 a_{10}}\omega_2^2 b_{10}^2+\frac{\gamma_1}{4\omega_1 a_{10}}\omega_2^2 b_{10}^2\cos 2\varphi_{20}-\frac{\gamma_3}{2a_{10}}\omega_2 b_{10}^2\cos 2\varphi_{20}$$

$$+\frac{f_1}{2\omega_1 a_{10}}\cos\varphi_{10})a_{11}-(\frac{1}{\omega_1}\omega_2^2\gamma_1 b_{10}-\frac{1}{2\omega_1}\omega_2^2\gamma_1 b_{10}\cos 2\varphi_{20}+\omega_2\gamma_3 b_{10}\cos 2\varphi_{20}$$

$$+\frac{3\gamma_4}{4\omega_1 a_{10}}\omega_2^2 b_{10}^2\cos\varphi_{20})b_{11}-(\frac{2\gamma_1}{4\omega_1}\omega_2^2 b_{10}^2\sin 2\varphi_{20}-\gamma_3\omega_2 b_{10}^2\sin 2\varphi_{20}$$

$$-\frac{\gamma_4}{4\omega_1 a_{10}}\omega_2^2 b_{10}^3\sin\varphi_{20})\varphi_{21}-(\frac{f_1}{2\omega_1}\sin\varphi_{10})\varphi_{11} \tag{51}$$

$$b_{11}^{'}=(-\frac{1}{4\omega_2}\omega_1^2\gamma_2 a_{10} b_{10}\sin 2\varphi_{20})a_{11}-(\frac{1}{2}\mu_2+\frac{1}{8\omega_2}\omega_1^2\gamma_2 a_{10}^2\sin 2\varphi_{20}$$

$$+\frac{9}{8}\omega_2^2 b_{10}^2 G_2)b_{11}-(\frac{1}{4\omega_2}\omega_1^2\gamma_2 a_{10}^2 b_{10}\cos 2\varphi_{20})\varphi_{21} \tag{52}$$

$$(\varphi_{21}^{'}-\frac{\varphi_{11}^{'}}{2})=(\frac{\gamma_2}{2\omega_2}\omega_1^2 a_{10}-\frac{\gamma_2}{4\omega_2}\omega_1^2 a_{10}\cos 2\varphi_{20})a_{11}+(\frac{\sigma_2}{b_{10}}-\frac{\sigma_1}{2b_{10}}+\frac{\gamma_2\omega_1^2 a_{10}^2}{4\omega_2 b_{10}}$$

$$-\frac{\gamma_2 a_{10}^2}{8\omega_2 b_{10}}\omega_1^2\cos 2\varphi_{20})b_{11}+(\frac{\gamma_2 a_{10}^2}{4\omega_2}\omega_1^2\sin 2\varphi_{20})\varphi_{21} \tag{53}$$

The stability of a particular fixed point with respect to perturbations proportional to $\exp(\lambda t)$ depends on the real parts of the roots of the matrix. Thus, a fixed point given by equations (50)-(53) is asymptotically stable if and only if the real parts of all roots of the matrix are negative.

4. Numerical results

The behavior of the given system of equations (1), (2) has been solved numerically applying Runge-Kutta 4th order method [20, 21]. Fig. 1 illustrates the response for the non-resonant system at some practical values of the equations parameters. From this fig. we can see that the system is stable with the steady state amplitude x and y are 0.05 and 0.3 respectively.

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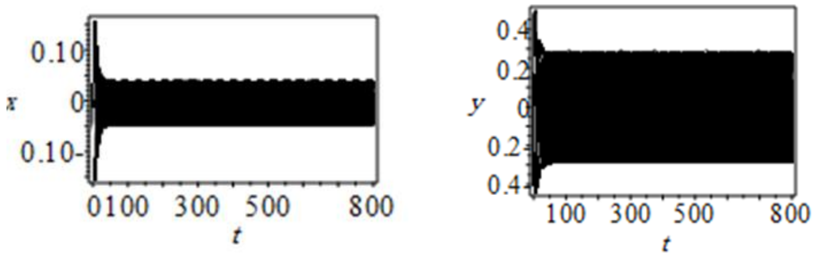

Fig. 1: The basic case of the system without controller.

(ω_1=2.4, ω_2 = 3.3, Ω = 2.3, f_1 = 1.7, f_2 = 1.6, μ_1 = 0.53, μ_2 =0 .17, γ_1=0 .19,
γ_3 =0 .9, γ_4 = 1.61, γ_2= 2.16, β_1= 0.023, β_2 =0 .12, β_3 = 0.05)

4.1 Resonance Cases

Some of the deduced resonance cases of the plant without the controller are studied numerically, we see that the amplitude increasing at the resonance cases and the worst case is the simultaneous resonance case when $\Omega = 2\omega_1, \omega_2 = \omega_1$, which the amplitudes are increased to about 2700 % and 66.7 % compared with the basic case shown in fig. 1. It can be shown that the amplitudes x and y are increasing to 1.4 and 0.5 respectively compared with the system without controller shown in Fig.2, which means that the system needs to reduce the amplitude of vibration or controlled, in Fig. 3.

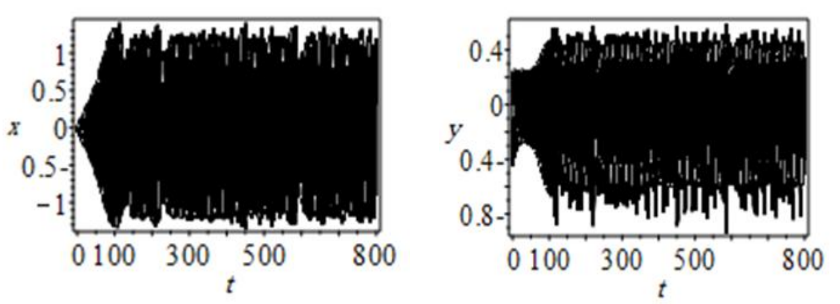

Fig. 2: System behavior without controller at simultaneous resonance

4.2 Effect of the Controller

Fig. 3, illustrates the results when the controller is effective, when $\Omega = 2\omega_1, \omega_2 = \omega_1$. The effectiveness of the controller is Ea (Ea =steady state amplitude of the main system without controller/

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steady state amplitude of the main system with controller) are about 56 and 3.8. The amplitude of the system is monotonic decreasing function of the gain coefficient G_1 and G_2, as shown in Fig. 4.

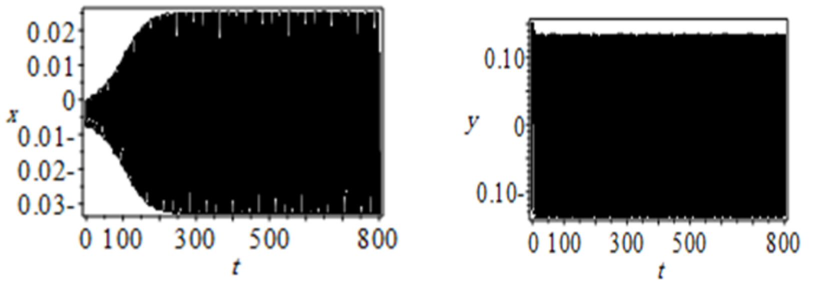


Fig. 3: System behavior with controller at simultaneous resonance

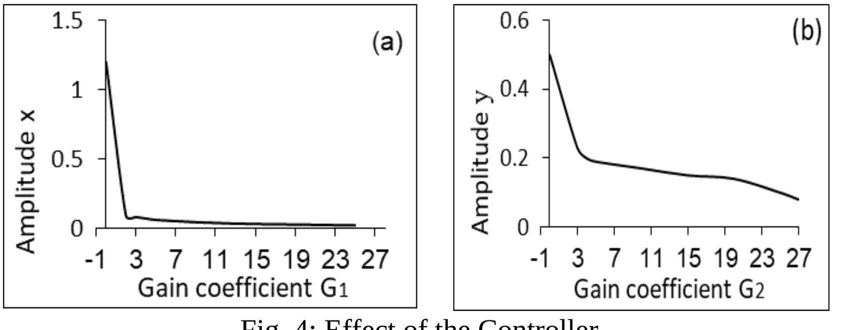
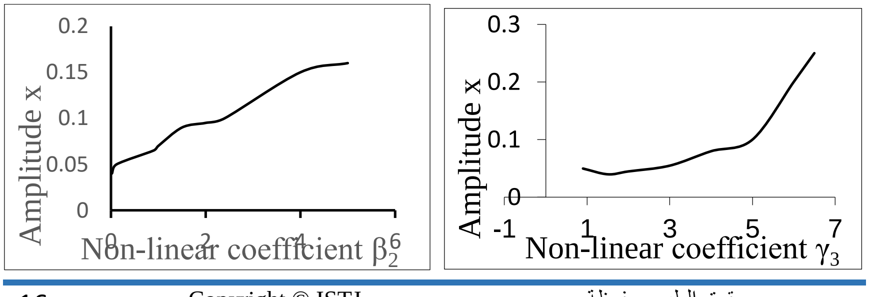


Fig. 4: Effect of the Controller

4.3 Effect of Parameters

The amplitude of the x is monotonic increasing function of the non-linear coefficient β_2, γ_3, γ_4 and the excitation force f_1, but amplitude of the system is monotonic decreasing function of the damping coefficient μ_1 as shown in fig. 5

The amplitude of the y is monotonic decreasing function of the damping coefficient μ_2, but amplitude of the system is monotonic increasing function of the excitation force f_2 as shown in fig. 6.



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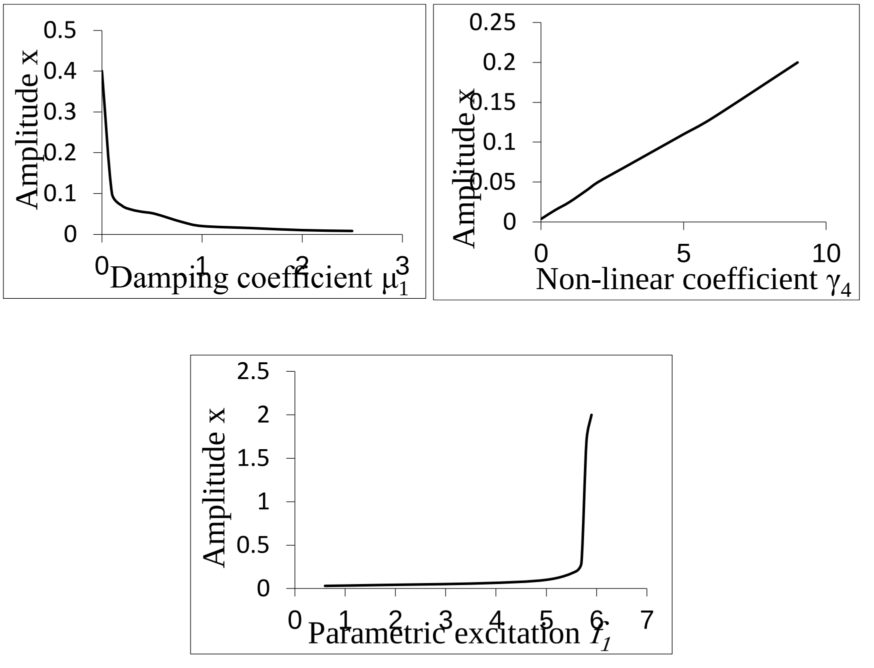

Fig. 5: Effect of Parameters

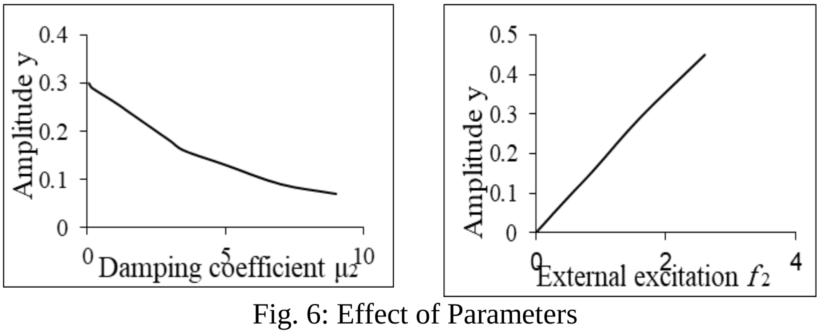

Fig. 6: Effect of Parameters

4.4 Response curves:

The frequency response equations (34) and (35) are nonlinear algebraic equations of a, b. These equations are solved numerically as shown in Figs. 7, 8.

Fig. 7, shows that the steady state amplitudes of the system are monotonic increasing functions in ω_1, γ_3, and monotonic decreasing functions in μ_1, γ_1.

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Fig. 8, shows that the steady state amplitudes of the system are monotonic increasing functions in γ_2 and monotonic decreasing functions in ω_2, G_2.

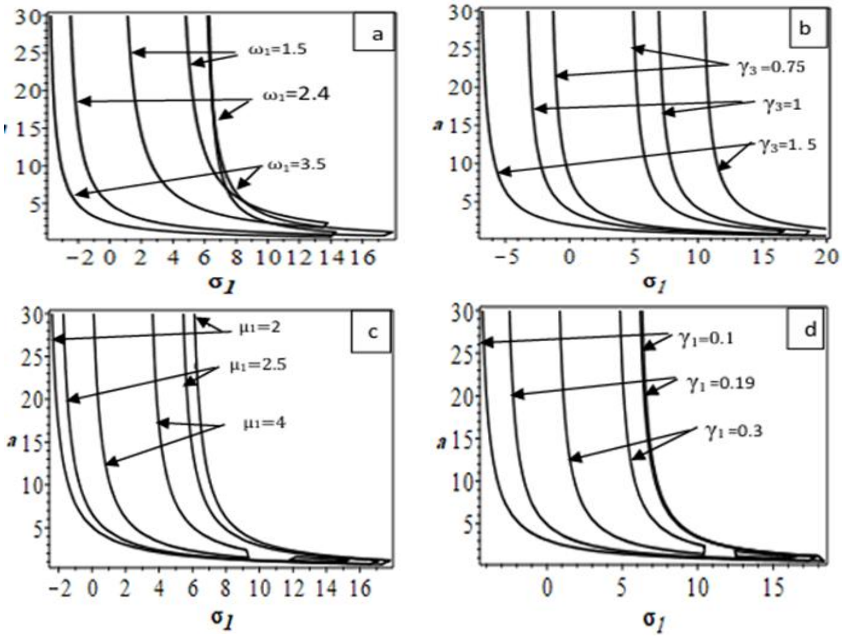

Fig. 7: Response curves (a≠0 and b ≠0)

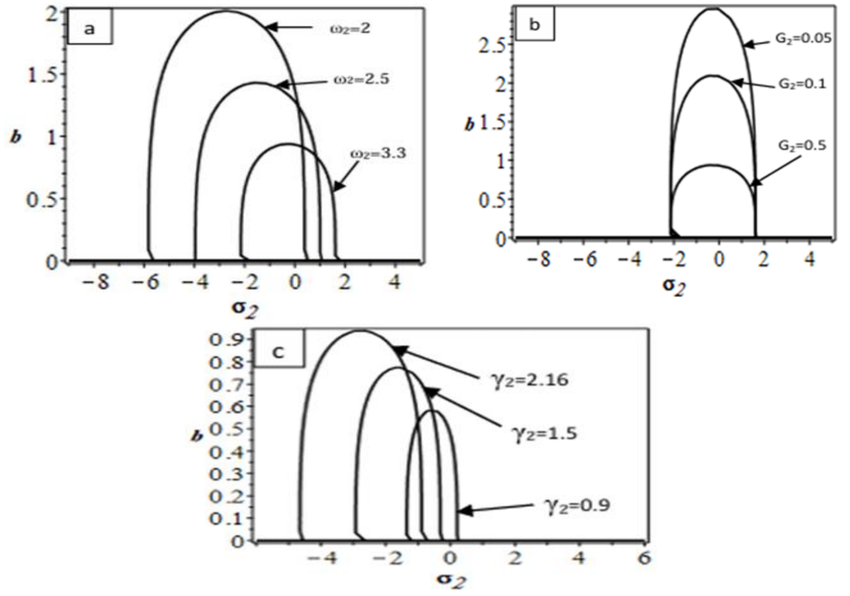

Fig. 8: Response curves (a≠0 and b ≠0)

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5. Conclusion

The vibrations of a coupled second order nonlinear differential equations having

1- The worst resonance case is the simultaneous resonance case when $\Omega = 2\omega_1, \omega_2 = \omega_1$ which the amplitudes are increased to about 2700 % and 66.7 % compared with the basic case.

2- The control can reduce the amplitudes x and y to about 0.025 and 0.13 respectively compared with the system without control.

3- The amplitude of the x is monotonic increasing function of the non-linear coefficient β_2, γ_3, γ_4 and the excitation force f_1, but it is monotonic decreasing function of the damping coefficient μ_1.

4- The amplitude of the y is monotonic decreasing function of the damping coefficient μ_2, but it is monotonic increasing function of the excitation force f_2.

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